Evolutionary Game Theory in financial market microstructure

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Research Description : Evolutionary game theory and its applications.

Research Outline

Recently, the financial regulation reform has been discussed by the lessons from the subprime crisis and the Lehman Shock that triggered the global financial crisis. What kind of regulations is better? To answer this question, I have developed methodologies to understand a financial market microstructure with evolutionary game theory and have made systems to use the developed methods in practice.

My research is focused on mainly on evolutionary game theory. There are gaps between the prediction of the theory and real economic phenomena. This is called an anomaly in economics. I have explored models which are consistent with real economic phenomena. I explained the anomaly though several proposed theoretical models which have been applied to real experimental data.

In detail, my research focuses on the similarity between the discreetness in number of strategies in game theory and the choice model in microeconometrics. I have provided estimates for the player's utility and have also predicted the player's behavior. In view of the fact that the classical game theory cannot be applied in a straightforward manner to the present study in game theory, I have developed extensions to the classical game theory that apply to the study. For example, from the statistical mechanics viewpoint, I formulated a model that accommodates large population of players in the game. I extended risk-neutral to risk-attitude of each player's utility. Also, I found that each player's behavior will be irrational if each player has long memory. Having extended the validity of the classical game theory, I studied some problems in financial market using experiment data.

The developed methodology can be adapted in practice. For example, the developed methodology have been applied the order book, to predict the execution price variation. The financial market microstructure can be considerable as advancement in financial technology, as it allows the use of high frequency data in finance.

Empirical study of the national bond price estimation model using market data

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Research Description : Empirical study on the cross-sectional market (CSM) model to estimate government bond price from market data

Research Outline

With the added burden on the national treasury expected to be accumulated due to the sluggish economic growth, sluggish tax revenue, and growth of the aging society with declining birth rate, it is less likely that the roles of the national bonds will be reduced in Japan.

This study aims to implement an empirical study of the national bond pricing model developed by Kariya (1995), viz. the CSM Model, and to examine the expansion etc. of models based on the study findings in light of the convenience of national bonds in properties probably with no credit risk. The characteristics of this CSM model include the considerations for the price difference according to the national bond attribute information (coupon (rate), redemption period, etc.). The price difference includes what is called “coupon effect” and “redemption period effect” that are considered to be important factors in explaining the transaction prices in the market. The CSM model considers the theoretical price of national bonds as a clustering of future fixed cash flow that is converted to the prevent value by a attribute-dependent stochastic discount function at the point of accrual. The model formulates the attribute-dependent stochastic discount function by incorporating the above-mentioned attribute information likely to impact on the national bond prices (difference in the coupon size and the difference of the length before redemption, etc.) in a part of explanatory variables and estimate parameters using market price data by the generalized least squares method. In this occasion, correlations at the accrual of the cash flow expressed in a variance-covariance matrix will be used as well.

The CSM model has been often used as a pricing model for practice without proper understanding of the national bond price, which is a random variable. However, the current study will enable the theoretical prices of national bonds to be reasonably derived.

I have completed the formulation for examination and the construction of the system that will serve as a base for examination, and started an empirical study. With the objective of providing a model with higher accountability for the market price, I am advancing demonstration, improvement, and comparison with existing general study models.

In addition, I am also working on the validation of the sizes of impacts, such as coupon effect and redemption period effect of national bond in market transaction, and of the changes in yield rates (yield curve).

Study on the spiral dynamic coexistence state of three competitive species

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Research Description : Pattern formation mathematics in the reaction-diffusion system

Research Outline

In fiscal 2009, the reporter implemented a study focusing on numerical experiments by a model equations using the reaction-diffusion system in order to confirm the appropriateness of the argument that "coexistence in the natural world is enabled by the complicated networks resulting from complicated competition etc. of more than two species that mitigates the strong competitive relationship."

The study background will be introduced below before discussing the research description.

Interactions among populations in the ecosystem include competitive relations such as a fight over the same resource. A Russian ecologist Gause proposed the "competitive exclusion principle" that individuals with strong interspecific competitiveness will not coexist based on a laboratory-level observation of protozoans. In the natural world, however, coexistent populations with intensive interspecific competition are observed. "Spatial segregation," "temporal segregation," and "competitive relaxation caused by the complication of interspecific relations due to interactions among many species" were included in the possible reasons for the fact and were conventionally studied so far.

The reporter particularly studied the case where spatial distribution has an important impact of the "competitive relaxation by invasion of a weak competitive species."

The model equation system, which the reporter studied, is a reaction-diffusion system in the two-dimensional space on a bounded convex domainΩ, with reaction boundary conditions given as boundary conditions and proper nonnegative functions given as initials conditions.

$$\frac{\partial}{\partial t}u_i = d_i \Delta u_i + f_i(u_1, \cdots, u_N), t > 0, x \in \Omega, (i = 1, \cdots N)$$

The nonlinear term f is limited to the following equation:

$$f_i(u_1, \cdots, u_N) = (r_i - a_i u_i - \sum_{i \neq j}^{n} b_{ij} u_j) u_i$$

It is known that the competitive exclusion principle holds in the case of N=2 of this system. In addition, it is known that, if this model equation is handled in the one-dimensional infinite interval given N=2, a traveling-wave solution is included that indicates expulsion by one species of the other species. In the meantime, if N=3, where three competitive species have equivalent strength, and further if the diffusion effects is disregarded, Ei Ikota and Mimura (1999) discovered a case that, despite the fact that competitive expulsion holds, coexistence will be enabled by the dynamic pattern formed in the segmented region as a result of the introduction of the diffusion term, and that the competitively balanced three traveling waves corresponding to the above-mentioned traveling waves given play an important role in the mechanism.

In terms of the hypothesis that an invading species cannot survive without consideration of the diffusion effect, a coexistence state has been numerically discovered in recent years where an invading species weaker than the existing two species survives with the other two species as a result of generation of a dynamic spiral pattern.

The reporter advanced numerical experiment with attention paid to the relations with the traveling wave given the one-dimensional infinite interval as well as reading previous studies extensively in order to deepen the understanding of this dynamic spirally-shaped coexistence state. As a result, it was demonstrated that the mechanism of this dynamic coexistence state is one that cannot be interpreted from the relations of the traveling wave given N=2 unlike the coexistence state in the case of Ei Ikota and Mimura. The detailed mechanism leading to coexistence remains a future task.

A part of the study result was presented at the Japan-Taiwan Joint Workshop for Graduate Students in Applied Mathematics held at the end of February 2010.

Development of a method for measuring advertising effects using Design of Experiments and time series analysis

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Research Description : Study on advertising effects on brand sales and values using time series analysis

Research Outline

I work on the study of measurement of advertising effectiveness, particularly with the study of “Extraction of Effects of Several Media Including the Internet Media” and “Long-term Effects of Advertising,” which are hot themes among others. In addressing those issues, I attempted to apply Design of Experiments to the former subject and attempted to apply time series analysis based on the theory of KM_2O-Langevin equations to the latter subject. The result of the former was presented at the 39th Study Assembly of the Japanese Society for Quality Control (October 31) and the result of the latter was presented at the “Complex Phenomena Time Series Analysis” Symposium (November 19) of the Meiji University GCOE program【Development of the Mathematical Sciences Based on Modeling and Analysis for Nonlinear Time Series】.

While these two approaches finally aim to measure and extract the effectiveness of advertising, they are significantly different in terms of the results to be obtained and the data to be used. While Design of Experiments attempts to measure by comparing the subjects divided into the control group and the experiment group to verify a certain hypothesis, the time series analysis attempts to derive findings and hypotheses from sales data and so on with hypothesizing avoided as much as possible.

Using Design of Experiments, a questionnaire survey was conducted on the web. The subjects were asked to look at the advertising and asked to answer to questions about the advertised product (such as favor and intention for purchase of the product) before and after seeing the advertising. On this occasion, the type of advertising was determined by the subject based on Design of Experiments. To treat with sampling errors, I proposed to apply “Hierarchical Bayesian Generalized Linear Model” instead of the conventional Analysis of Variance. In addition, this model enabled simulation for minimization of advertising budget and so on.

Using time series analysis, applying Self-Organizing map, I proposed a method that visualizes changes in non-linear dynamics by calculating the quantities relating to non-linear dynamics based on the theory of KM_2O-Langevin equations for the product sales data (POS data). Most of preceding studies detect such changes using linear models that cannot discriminate temporal outlier from changes in non-linear dynamics. Whereas, this study enables discrimination of temporal outlier from changes in non-linear dynamics because it uses nonlinear models. In addition, I examined the relation between the change point and the timing of advertising exposure including advertising creative. This method is expected to be applicable not only to marketing but also various phenomena.

Evolution of phenotypic traits in a predator-prey community

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Research Description : Elucidation of biological systems by Mathematical Modeling and Analysis

Research Outline

Evolution through natural selection is often understood to imply improvement and progress. A heritable trait that has a higher fitness will spread within the population. The average fitness of the population would therefore be expected to increase over time. However, this paradigm neglects the evolutionary mechanism: Although the environment selects the adaptations, these adaptations will change the environmental conditions. By moving across a fitness landscape, populations change that landscape, new peaks and valleys form, channeling its further motion. The fitness landscape is shaped by the phenotypic distributions of the involved populations. This viewpoint affects not only the intuition of evolutionary biologists but also their theoretical tools. The theory of adaptive dynamics is an appropriate framework developed for understanding the long-term evolutionary outcomes of small mutations in the traits expressing the phenotypes. If the environmental conditions necessarily coevolve, then the spectrum of possible dynamical behavior becomes a lot richer.

In order to understand the mechanisms of evolutionary diversification and evolutionary extinction, in the last year, I mainly investigated the evolution of phenotypic traits in a predator-prey system subject to Allee effect. With the methods of adaptive dynamics and bifurcation analysis, we investigated the influence of Allee effect on the evolution of phenotypic traits of predators and prey. Firstly, we identified the ecological and evolutionary conditions that allow for continuously stable strategy and evolutionary branching. It was found that prey population undergoes evolutionary branching if the Allee effect of prey population is not strong and the frequency dependence in the competitive interactions and predation efficiency is strong. Secondly, we investigated the conditions that allow for evolutionary suicide and evolutionary cycle. We found that evolutionary suicide occurs deterministically on prey population if prey individuals undergo strong asymmetric competition and are subject to Allee effect. Moreover, by using Hopf bifurcation theorem, we showed that evolutionary cycle is a likely evolutionary outcome, which depends on the strength of Allee effect and the mutation rates of predators and prey. The analysis revealed that how and why prey population becomes extinct during the course of evolution. This paper has been published in Journal of Theoretical Biology (2010, 262 (3): 528-543).

In addition, why and how specialist and generalist strategies evolve remain the important questions in evolutionary ecology. In the last year, I also investigated the evolution of foraging-related traits in a predator-prey community with trade-off structure. With the methods of adaptive dynamics and geometrical argument, first, we identified the ecological and evolutionary conditions that select for specialist and generalist strategies. Generally, generalist strategy evolves if there is a switching benefit; if there is a switching cost, both specialists do better than the generalist. Second, we found that if the trade-off curve is globally convex and the switching cost is large, predators always evolve to the closest specialist strategy. However, if the switching cost is small, evolutionary branching in the predator phenotype occurs, predator population eventually branches into two extreme specialists, each completely specializing on a particular prey species. Third, it was found that after branching in predator phenotype, if the trade-off curve is concave-convex-concave, then there exists an evolutionarily stable dimorphism in which two predators can continue to coexist on two competing prey on the long-term evolutionary timescale. The analysis reveals that an attractive dimorphism will always be evolutionarily stable and that no further branching is possible under our model.

Overall, during the past year, I have done some interesting work. I greatly improved my research ability and learned a lot of methods and techniques to deal with the core problems in evolutionary biology.