Quantum walk
—moving around in the quantum world—

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Specialized Field, Academic Degree	:	Discrete Geometrical Analysis, Ph.D., The University of Tokyo
Research Description	:	Analysis of Network System

Research Outline

A random walk is a stochastic process defined on a network (graph) that has been studied since the last century with various applications incorporated. In particular, it is known that various limit theorems hold for a random walk on a topological crystal (crystal lattice) (e.g. central limit theorem, local limit theorem, large deviation properties). The Kotani-Sunada's theory revealed that the limit theorems relate to the structure of the topological crystal when viewed with a geometric standpoint. For example, the special realizations with the maximum symmetry of topological crystals (called the standard realizations) are deeply related to the central limit theorem. The method developed by Sunada to prove the limit theorems is what we call Discrete Geometric Analysis.

The quantum walk is a “quantum version” of random (classical) walk. While the classical walk needs “external” randomness as in coin throwing, the quantum walk is a walk caused by the probabilistic aspects of the quantum mechanics. This field has been significantly developed in recent years with the study of quantum computers. A quantum walk on the 1-dimensional lattice is defined by discretizing solutions of the 1-dimensional Dirac equation. Thus the probability of moving to the right or left is replaced with a two-by-two matrix acting on the spinor. Accordingly, a quantum walk is regarded as a “non-commutative version” of the classical walk. As revealed in the study by N. Konno, the transition probability distribution for the 1-dimensional quantum walk converges in law to a certain continuous probability distribution (under proper scaling).

In this study with T. Tate, a partial refinement of the Konno's finding was given. While this may be considered as similar to the large deviation asymptotic behavior in the case of the classical walk, the result shows that the asymptotic behavior for the quantum walk is utterly different from the classical walk.

In addition, it was revealed, given a proper definition to the quantum walk on a general graph, that a quantum walk with an apparently “non-commutative” aspect is actually dealt with in a “commutative world.” In other words, a vector-valued quantum walk can be reduced to a scalar-valued quantum walk. Further, I studied asymptotic behaviors of a quantum walk on a general topological crystal from the view of discrete geometric analysis and succeeded in deriving a partially generalized result (I am in the process of writing a paper). However, with many problems remaining unsolved, I am still continuing the study.

Mathematical analysis of self- organization

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Specialized Field, Academic Degree : Mathematical Modeling and Analysis (MMA), Ph.D., Kyoto University

Research Description : MMA of Nonlinear Non-equilibrium Phenomena

Research Outline

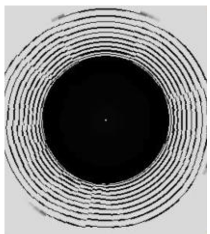
The term, self-organization, was used for the first time in 1947 by R. Ashby who is a psychiatrist as well as a pioneer in cybernetics and then mathematical models with the concept incorporated appeared in 1952. A Mathematician in England, A. Turing, explained the morphogenesis mechanism, which was an unsolved problem among the biological phenomena of those days, using a simple differential equation and set forth a diffusion paradox that “diffusion-induced spatial instabilities”. This theory suggests that biological phenomena are not always caused by a genetic control from top down but may spontaneous occur if there are diffusion of several form factors and proper balance between interactions and is an idea from a self-organization perspective. This idea, however, was not accepted in the biological community in those days when molecular biology and molecular genetics were just born. The reason for unacceptability was that the model he used was not the description of real biological systems but was a “metaphor” for extraction of the essence only. In the same year, neurophysiologists in England, A. L. Hodgkin and A. Huxley proposed an ionic theory for a membrane potential pulse wave that propagates on the nerve axon at a constant rate and derived a mathematical model based on their theory. Their model could not be solved by analytical methods at that time; however, fortunately for them, a large scale high-speed computer that appeared in those days indicated that the model successfully reproduced the pulse wave observed in the experiment. In this way, the second diffusional paradox “diffusion generates waves by coupling with reactions” was born. Later, various complex phenomena appearing in the natural and social science fields came to reveal that those paradoxes are the essential mechanism causing self-organization. My research started with the above as a background. Motivated by the self-organization phenomena, I published the following papers in fiscal 2009.

[1] X.-C. Chen, S.-I. Ei and M. Mimura: Self-motion of camphor discs: Model and analysis, Networks and Heterogeneous Media 4, 1-17 (2009)

[2] D. Hilhorst, R. van der Hout, M. Mimura and I. Ohnishi: A mathematical study of the one dimensional Keller and Rubinow model for Liesegang bands, J. Statistical Physics, 135, 107-132 (2009)

[3] D. Hilhorst, M. Mimura and H. Ninomiya: Fast reaction limit of competition diffusion systems, to be pressed in Handbook of Differential Equations:Evolutionary Differential Equations, vol. 5, eds. C. Dafermos and M. Pokorny, Elsevier, 105-168 (2009)

Liesegang spiral that appears in the experiment



Liesegang spiral reproduced by simulation

Theoretical Foundation of Combinatorial Optimization and Application to Mathematical Sciences Based on Modeling and Analysis

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Position Title,Affiliated Department : Fellow, MIMS; Professor, School of Science and Technology, Meiji University

Specialized Field, Academic Degree : Theory of Computation, Ph.D., University of Tronto

Research Description : Computation and Theory of Algorisms

Research Outline

Combinatorial optimization was studied broadly from theoretical foundation to applications.

For theoretical approaches, the researcher emphasizes studies on branch decomposition of graphs as the basis for very large-scale neighborhood local search. The overall structure of the study is as described in my report in the preceding fiscal year's report. In fiscal 2009, I focused my effort on studies on approximation algorithms for branch decomposition of planar graphs and a new upper bound on the ratio of the branch width of a planar graph to the size of its maximum grid minor together with Professor Qianping Gu of Simon Frasor University. It was demonstrated that, when the branch width of graph G is indicated as bw (G), and the maximum value of g where G has a g × g grid minor is indicated as gm (G), an equation bw(G) ≦3gm(G) + 1 holds for the planar graph G. This is an improvement from the following inequality proposed by Robertson, Seymour, and Thomas: bw(G) ≦4gm(G) + 1. This improvement is important as the constant in the right side appears in the exponent of many functions representing the execution time of many algorithms used in this inequality. As for this result, I have summarized the details of the demonstration in two parts of technical report, and I will contribute the combined report to an international conference. In addition, I have developed a constant approximation algorithm for O (n1 + ε) time by probing deeply the compositional demonstration of the above inequality in algorithm terms with respect to both the problem of obtaining the minimum width to the planar graph and the problem of obtaining the largest grid minor. This result was presented in the International Symposium on Algorithms and Computation held in December 2009.

In applications, I have been working on computer science applications such as for graphic drawing. I have also been studying applications in mathematical sciences based on modeling and analysis, particularly for simulations. As for the problem of removing the 180-degree uncertainty from the solar magnetic fields described in my report in the preceding fiscal year, I completed a program in the very large-scale neighborhood approach, and confirmed that it can reduce the value of the objective functions considerably relative to the sample input. The full-scale evaluation remains to be done. Further, I studied whether branch decomposition is applicable to the analysis of Boolean networks that has been studied as a gene network model. As a result, although I could not find out a method of directly applying branch decomposition, I found some cases where the analysis, which has been conventionally only possible through sampling by simulation, can be performed strictly using a related concept, viz. directed path-decomposition. I am in the process of organizing this approach as a research paper.

Further, as an issue independent from branch decomposition, I studied a subject of directing each side of a non-directed graph to a single direction to keep the directed distance between vertices as small as possible. This subject was selected with the method of setting the contraflow traffic in factories and markets as a motivation. A part of this result was presented at the International Symposium on Algorithms and Computation held in December 2009.

Mathematics of diffusion/propagation phenomena and pattern structures

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Position Title,Affiliated Department	:	Fellow, MIMS; Associate Professor, School of Science and Technology, Meiji University
Specialized Field, Academic Degree	:	Nonlinear Partial Differential Equations, Ph.D. (Science), Kyoto University
Research Description	:	Mathematical science of diffusion and propagation phenomena and pattern structures

Research Outline

Chemical particles move randomly. As we often observe, their distributions become homogenized. This phenomenon is called “diffusion”. The diffusion effect is, however, known to play various roles together with reactions. In 1952, A. Turing discovered that the diffusion can create periodical patterns, which is called “diffusion-induced instability” or “Turing's instability”. This is an important concept as a mechanism of pattern formation.

In some equations, solutions become infinity within a finite time. We call it a “blowup” of the solution. In 1998 we found the system where a reaction-diffusion system possesses a blowing-up solution, while all solutions of the corresponding ordinary differential equations without diffusion converge to the origin. We call this paradoxical phenomenon a “diffusion-induced blowup”. We are trying to characterize the reaction term (nonlinearity) where the diffusion-induced blowup takes place.

On the other hand, propagation phenomena are also caused by the interaction of diffusion and reaction. We may often observe an epidemic or the propagation of chemical waves. As seen in the invasion of the epidemic, the speed of the propagation is uniquely determined. We call this solution which moves with a constant speed a “traveling wave”. Now I am studying the traveling wave in the multi-dimensional space. Since the spatial patterns of traveling waves play an important role, we are constructing various types of traveling waves, like V-shaped, finger, zipping wave and so on.

As stated above we are investigating the structure of nonlinearity and diffusion by studying diffusion from various viewpoints.

1. D. Hilhorst, M. Mimura, H. Ninomiya: Fast Reaction Limit of Competition-Diffusion Systems. In: C.M. Dafermos and Milan Pokorny, editors: Evolutionary Equations, Vol 5, Handbook of Differential Equations, Hungary: North-Holland (2009), 105—168
2. J.-S. Guo, H. Ninomiya and J.-C. Tsai: Existence and uniqueness of stabilized propagating wave segments in wave front interaction model, Physica D: Nonlinear Phenomena 239 (2010) No. 3-4, 230—239