

3. Program members and their research outlines

FY2008

Meiji University Global COE Program



Formation and Development of Mathematical Sciences Based on Modeling and Analysis

MODELING GROUP

<i>Group Leader</i> Yasunori OKABE	Development of Experimental Mathematics: Theory of Stochastic Processes and Time Series Analysis, Theory of KM2O-Langevin equations and Theory of Soliton, Dirichlet Forms and Discrete Convex Analysis
Masao MUKAIDONO	Mathematical Modeling and Analysis of Fuzzy Systems
Takeaki KARIYA	Financial Engineering, Mathematical Finance, Value-Creating ERM
Hiroyuki MORI	Intelligent Systems, Data Mining, Meta-heuristics, Power Systems Engineering, Power Market Analysis, Distribution System Automation
Ryo KOBAYASHI	Mathematical Modeling and Analysis of Self-organization
Kaoru ARAKAWA	Perceptual information processing in consideration of human mind, Understanding human mind by biological signal analyses, Image and sound processing, Information theory
Hiraku NISHIMORI	Mathematical Modeling and Analysis of Cooperative Phenomena
Tatsuo SHIBATA	Mathematical Modeling and Analysis of Bio-network Systems
Joe Yuichiro WAKANO	Mathematical Biology: Evolution of social learning and individual learning, Evolution of cooperation, Pattern formation in spatial games

MATHEMATICAL ANALYSIS GROUP

<i>Group Leader</i> Toshikazu SUNADA	Discrete geometric analysis and its applications
Masayasu MIMURA	Mathematical Modeling and Analysis of Nonlinear Non-equilibrium Phenomena
Hisao TAMAKI	Computation and Theory of Algorithms

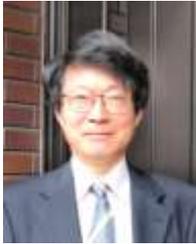
SIMULATION GROUP

<i>Group Leader</i> Kanya KUSANO	Modeling and Simulation of Large-scale Cascade Systems
Kazushi AHARA	Development of software for researchers of topology
Daishin UEYAMA	Mathematical understanding of the complex pattern formation in dissipative systems, Simulation aided mathematical analysis, Effective visualization for scientific computations

GCOE RESEARCH FELLOWS

Kota IKEDA	Reaction-diffusion equation, Pattern formation
Wataru NAKAHASHI	Theoretical Anthropology: Evolution of learning ability, Sexual selection

Development of Analysis Technology for Capturing Transition in Dynamics of Complex System Time Series



Yasunori OKABE *Modeling Group Leader*

Position Title, Affiliated Department : Fellow, MIMS; Professor, School of Science and Technology, Digital Content Studies and the Connective Humanities, Meiji University
Specialized Field, Academic Degree : Stochastic Analysis and Time Series Analysis, Ph. D.
Research Description : MMA of Time Series Data

Research Outline

The phenomena of complex systems that are subject to time-series analysis from the standpoint of experimental mathematics are not those where the time-series model is known from physical principles. They are rather physical phenomena, including deep low-frequency earthquakes, where the time-series model is unknown. In the time-series analysis under the current study, models are not given by default to which data can simply be plugged in, but the models hidden deep in the data are derived as the necessary condition. The current study thus features a "from data to model" research approach. The information obtained through verification, using the data only, of hypotheses for the proposition to be applied to the time-series analysis is regarded as the "first discovery." The objective is to identify the "second discovery," which is hidden deep in the phenomena of the complex system, by studying the mathematical structures of the first discovery, thus bridging the phenomena and mathematics.

When stationarity of a complex system time series is detected in Test(S) with a time-series analysis based on the theory of KM2O-Langevin equations, an equation describing temporal evolution of the time series can be derived. The dissipation term of the formula has a polynomial, nonlinear structure. In particular, the objective of the study is to determine, upon detection of the time series abnormality using Test(ABN) (a function designed to identify such abnormality), what transition the nonlinear structure of the dissipation term in the stationarity time zone after the point of time of the abnormality is undergoing from that in the stationarity time zone prior to the same point of time of the abnormality.

- (i) Four years ago, based on the theory of KM2O-Langevin equations, a new characteristic, separation property, was identified in the stationary time zone immediately after the arrival of S-waves of deep low-frequency earthquakes. Furthermore, last year, it was shown that the separation property has a mathematical structure in which the information space having a nesting structure obtained from applying the even-order polynomial transformation is isolated from that obtained from applying the odd-order polynomial transformation.
- (ii) Decision analysis and causal analysis based on the theory of KM2O-Langevin equations demonstrated that during the period of zero interest-rate policy implemented by the Bank of Japan for the first time in the world, the time series of the currency circulation rate, defined as the ratio of money supply to GDP, has both stationarity and determinacy, and further that there is a causal relationship between money supply and GDP.
- (iii) Using dynamics analysis based on the theory of KM2O-Langevin equations, it was identified that the "quadratic nonlinearity" is applicable to share prices lacking abnormality during the period after Black Monday when stationarity was being restored. This was reported at an international conference in Bulgaria last year.

In order to have the general public know the achievements with the use of time-series analyses based on experimental mathematics, a series of eight lectures was held from April 16 to June 11, 2008 as a Liberty Academy Business Program of Meiji University as follows:

Title: Experimental Mathematical Study of Economic Time Series and Hannya Shingyo (Core Sutra of Perfect Wisdom)--Capturing Predictors of Abnormality in Complex Systems Time Series Phenomena and Derivation of Dynamics

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- [2] Yasunori Okabe, On a time series analysis for complex phenomena based upon the theory of KM2O-Langevin equations, International Journal of Pure and Applied Mathematics, Vol.49, No.3, 2008, 309-316

Study on Risk Assignment between Man and Machine in Ensuring Safety



Masao MUKAIDONO

Position Title, Affiliated Department : Deputy Director, MIMS; Professor, School of Science and Technology, Program in Safenology, Meiji University

Specialized Field, Academic Degree : Safety Study, Ph.D.

Research Description : Modeling and Analysis on uncertainty systems

Research Outline

When man and machine coexist such as in the case of next-generation robots, it is not possible to remove the risk of hazards completely. However, the level of risks that are tolerable and the roles that should be assigned to man and machine for lowering the risks are extremely unclear, and this is an obstacle in introducing useful machines to social life. The current study is geared to proposing ideas on safety standards for coexistence of man and machine, by modeling ambiguity in risk allocation to man and machine.

The ultimate objective is to establish the principle (Principle of Coexistence) for useful but not risk-free man-machine coexistence, for example, in the case of next-generation robots for nursing and welfare. At present, there are a large number of machinery and equipment, including medical devices that are useful but cannot be brought into the daily social life due to lack of standards and ambiguity of risk allocation. The object of the study is to help formulate the safety standards for machinery and equipment in order for man and machine to coexist, by clarifying ideas such as to what extent risks must be reduced before they are tolerable and where is the boundary between the roles of man and those of machinery in ensuring safety, by examining the allocation to man and machine of the roles for ensuring safety through modeling of risk-based safety.

The Government is also promoting the line of study, and the Ministry of Economy, Trade and Industry already issued the Safety Guidelines for Next-Generation Robots (1) under the chair of the author. Thereafter, in FY 2009, the New Energy and Industrial Technology Development Organization (NEDO) launched a 5-year "Project for Practical Application of Life Support Robots" for service robots related to people's daily lives, which is geared to establish the certification system for life-support robots and to support practical launch of robot businesses. Research has begun on establishing safety standards, verifying safety, and establishing assessment methods. (The author serves as the chair of the Review Committee).

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- [2] K. Sugihara and M. Mukaidono, Structured Safety System of Power Assistance Robot, Proceedings of 39th International Symposium on Robotics, pp.460—465, 2008-10
- [3] 向殿、安全技術の現代的課題と社会的受容性、精密工学会誌、Vol.75, No.9, pp.1041～1044, 精密工学会、2009-9
- [4] 杉原、向殿、安全設計の基本概念、品質、Vol.39, No.4, pp.7～15、品質管理学会、2009-10
- [5] 向殿、次世代ロボットの安全性、(基調講演) 産業・化学機械と安全部門研究発表講演会 2009 講演論文集、pp.1～5、日本機械学会、2009-11

Modeling and Management of Credit Risks



Takeaki KARIYA

Position Title, Affiliated Department : Fellow, MIMS; Professor, Graduate School of Global Business, Meiji University

Specialized Field, Academic Degree : Financial Technology, Ph.D.

Research Description : MMA of Finance

Research Outline

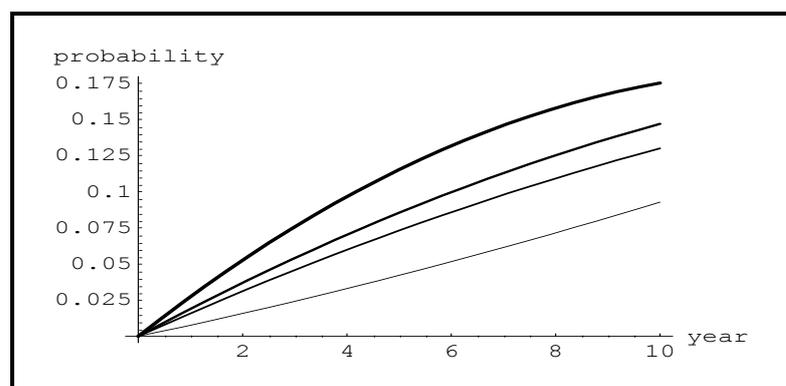
A bank's lending, as well as a company's sales receivables, involves credit risks. In order to manage these risks, it is essential to assess them appropriately. In the current study, the credit risk measurement method, which forms the basis for the assessment, is built around the model of credit risks of corporate bond prices proposed by Kariya (2007). In the method, a model is proposed to base the measurement on the following pieces of information: 1) corporate bond prices; 2) corporate bond ratings; 3) breakdown of the business portfolio by industry of the issuing company in terms of sales, etc.; and 4) government bond prices, and the term structure of credit risks is measured for each of the industry-rating combinations. In the current study, applications of the model are considered.

Based on the assumption that a bond issuing company is involved in a plurality of industries, the term structure of the probability of default and recovery rate is derived for each of the industry-rating combinations from many bond prices. Accordingly, the study will help price a newly issued bond having a different industry portfolio breakdown, as well as bank loans and derivatives. It will also help evaluate theoretical component values of these financial instruments with little liquidity. At the same time, it will provide a method for building a bond fund corresponding to a specific rating by combining various corporate bonds.

Unlike the traditional backward-looking model of estimating the probability of default from the historical corporate bankruptcy data, the model under study is forward looking, deriving the term structure of the probability of default and recovery rate for each of the industry-rating combinations from prices of many corporate bonds, on the assumption that investors are rationally seeing into the future in determining prices of bonds issued by existing companies. In addition, the discount function of government bonds used in discounting the future cashflows of the corporate bonds is dependent on the attributes of government bonds and is thus specific, derived from Kariya & Tsuda (1995).

Default rates of BBB-rated corporate bonds
From bottom to top:

1. Electric railway sector
2. Other sectors
3. Distributor/retailer sector
4. Construction/real estate sector



Studies on Optimization, Forecasts, and Rule Extraction for Electric Power Networks



Hiroyuki MORI

Position Title, Affiliated Department : Fellow, MIMS; Professor, School of Science and Technology, Electric Engineering, Meiji University

Specialized Field, Academic Degree : Intellectual Informatics, Ph.D.

Research Description : MMA of Intelligent Systems

Research Outline

The electric power industry is currently promoting the establishment of smart grids, as a means to fight global warming. This will result in large-scale introduction of distributed generation, such as wind power farms and mega solar plants, to the electric power networks. Accordingly, technologies that are excellent for operation and planning of large-scale, complex networks with uncertainties are in demand. The following three research themes are studied in the current project:

- 1) Optimization;
- 2) Forecasts; and
- 3) Extraction of rules.

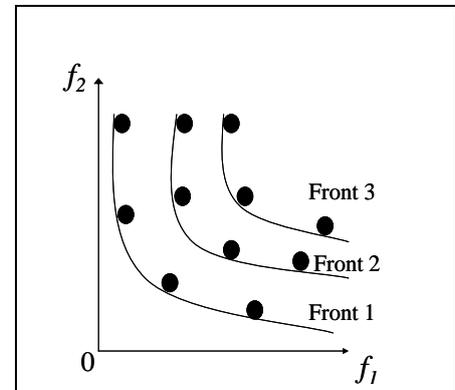


Figure 1: Fronts of Multi-purpose Optimization

In the first theme on optimization, the researcher is interested in metaheuristics, in which simple rules and heuristics are repeatedly applied to obtain a high-precision approximation of the global optimal solution. Subjects in the theme include the global optimal solution using metaheuristics; multi-objective evolutionary computation for systematic solution of the Pareto solution set using metaheuristics (see Figure 1); optimization of systems with uncertainties; hybrid metaheuristics; and algorithm for distributed parallel processing for use in optimization. The application studies of the first theme include expansion plans for power transmission and distribution networks; and distribution automation comprising optimization of network reconfiguration, control of power voltage and reactive power, control of recovery from accidents, state estimation, and load estimation. In the second theme on theoretical studies of forecast, the researcher is working on forecast models using a kernel machine in statistical processing of information. A kernel machine is used in building models in which the machine learns the model with linear sums of nonlinear functions. In application studies, such issues are being studied as power load forecast that is essential in power system operation planning; electricity price forecast in the electric power market; wind velocity forecast for wind power generation; emission rights price forecast in the CO₂ emissions trading market; solar radiation quantity forecast for solar power generation; and maximum temperature forecast that is closely related to the smart grids. In the third theme on theoretical studies for rule extraction, the researcher is studying rule identification with the use of a regression tree. It is a method for classifying complex data into terminal nodes with the use of “If-then” rules at branching nodes. The focus is on achieving high precision in identified rules, with application of fuzzy inference and ensemble learning to the “If-then” rules. The applications include clarification of the relationship between conditions of the power system and classification of indices for voltage stability of power systems; extraction of rules for power load forecast for operation of power systems; extraction of rules for power prices in the electric power market; and assessment of credit risk of players in the electric power market.

Fusion of Mathematical and Life Sciences



Ryo KOBAYASHI *Sub-leader of all researches*

Position Title, Affiliated Department : Fellow, MIMS; Professor, Department of Mathematical and Life Sciences, Graduate School of Science, Hiroshima University

Specialized Field, Academic Degree : Mathematical Modeling and Analysis, Ph.D.

Research Description : Mathematical study of structure formation, locomotion and information processing of living organisms

Research Outline

The themes of the current studies are as follows:

1. Control of systems with many degrees of freedom learned from locomotion of living organisms

In order to create robots that move around in a real, complex environment flexibly and lithely like living organisms, it is required to provide the body of the robots with many degrees of freedom that living organisms have, and control them with sophistication. Through a mathematical explanation of control mechanisms that generate the lithe movement, which is common to various forms of locomotions from ameoid to bipedal, the study aims to create a method for autonomous decentralized control of robots having many degrees of freedom.

2. Structural formation of living organisms and information

The study aims at creating a mathematical expression capable of describing structural formation of living organisms, and building a mathematical model that combines the information that regulates the structural formation. Specifically, the study has such themes as early cleavage processes, blood vessel formation in yolk sac, and formation of tubular network of plasmodium of *Physarum Polycephalum*.

3. Pattern formation of rivers

In nature, rivers seldom flow in a straight course: commonly observed patterns include meandering and braided streams. The object of the study is to reproduce formation of such patterns in the laboratory, and develop a mathematical model that explains the formation of these patterns in a unified manner.



Perceptual Signal Processing Considering Human Mind and Sense



Kaoru ARAKAWA

Position Title, Affiliated Department : Fellow, MIMS; Professor, School of Science and Technology, Dept. of Computer Science, Meiji University
Specialized Field, Academic Degree : Image and Speech Processing, Dr. of Engineering
Research Description : MMA of Perception Systems

Research Outline

Subjective evaluation, as well as quantitative evaluation, takes an important role in perceptual signal processing such as image processing. However, conventional methods of signal processing mostly take only the quantitative evaluation, such as minimum mean square error sense, into consideration. In order to build up more suitable signal processing for perceptual signals, a new method of signal processing is developed, which considers human mind and sense.

Specifically, an image processing system is developed for optimal noise removal and aesthetic modification considering human subjective criteria using interactive evolutionary computing (IEC). IEC is a method to optimize a system using the genetic algorithm (GA) with repetition of selecting survivors on the basis of human subjective criteria. Accordingly, IEC generates a system which is satisfactory to human sense. Moreover, IEC can easily optimize a nonlinear complicated system and can be applied to nonstationary signals, the mathematical model of which is hard to obtain. Especially, application of IEC to the face image beautifying system, which was proposed by this researcher before, successfully gives quite interesting performance; a system to beautify human face image, which considers the observers' subjective criteria and taste, is realized using IEC.

IEC can be also applied to any image restoration technique. For example, noise reduction of image, which is highly estimated by human vision, is realized by introducing IEC to conventional nonlinear edge preserving filters.

As to auditory signals, a new method of blind signal separation is proposed using GA. Blind signal separation is a well-known problem to separate mixed different signal sources, taking the independency among the signal sources into consideration. Various methods have been proposed for it, but these performance gets degraded, when random noise is mixed into the signals. However, high performance in signal separation is realized by GA, because GA is good to find out satisfactory solution, even if the evaluation function has local minima or is not sensitive to the system parameters.

Moreover, human mental states are examined by analyzing the electroencephalogram (EEG), when they read documents printed on a paper and those shown on PC display. It is clarified that people tend to feel more stress when they read smaller letters than larger ones, and especially the difference of the stress between the sizes of letters is larger, when they read them on PC display than on a paper.

Clarification of Dynamics and Functions of Moving Group of Elements



Hiraku NISHIMORI

Position Title, Affiliated Department : Fellow, MIMS; Professor, Department of Mathematical and Life Science, Graduate School of Science, Hiroshima University

Specialized Field, Academic Degree : Non-equilibrium Physics, Ph.D.

Research Description : MMA of Cooperative Phenomena

Research Outline

In the nature surrounding us, there are various types of groups: groups of fish, birds, and insects are well-known examples. In the human society, there are movements of various groups. The degree of flow or congestion of groups of automobiles or of pedestrians has substantial impacts on the productivity of the society in general. It has become known that, for the movement of dunes, or barchans, that cause huge damages to roads, pipelines and other man-made structures, the characteristics of temporal development of the system can be understood better by looking at the whole system as the movement of a group of many barchans. The researcher has been working on the issue, with particular emphasis on the following two points in FY 2008:

1. Development and analysis of new mathematical models for understanding the movement of barchans as a group movement; and
2. Development and analysis of flow models for groups of particles with application of chemotaxis of insects.

Concerning the first point, the researcher was successful in reducing the movement characteristics of barchans into a simple ordinary differential equation. Barchans are the type of dunes that move nearly independent of each other on hard base rock. The collision process of two or more barchans was also formulated as simultaneous ordinary differential equations. Further, the temporal development of the system according to initial conditions was classified as an orbit of a dynamical system, and qualitative agreement with experimental tests was demonstrated. This is a new trial for mathematically describing complex movements of dunes, which was not easy in the past, and it is expected that the model will develop into one that is applicable to more complex dune movements for forecasts. The results were published in academic journals, conference minutes, and a book of the researcher's writing.

Concerning the second point, the researcher incorporated factors of secretion, evaporation, and attraction of pheromones to the traditional mathematical model of pedestrians in two-way traffic, inspired by chemotaxis of ants. Accordingly, it was demonstrated that, depending on the density of pedestrians and evaporation quantity of pheromones, the pheromones reduced the congestion and increased the flow. Although it is not possible to apply the effects of pheromones directly to the pedestrians in the real world, the researcher believes that the concept of "embedding memories in the flow in the field" can be related to the characteristics of behaviors of individual pedestrians, such as the tendency to follow others walking in the same direction on a crowded road. Furthermore, one possible application is to incorporate the effects of pheromones to the automatic traffic system that could be introduced in the future in order to reduce congestion and increase the flow. Part of the results will be released in a forthcoming academic journal.

Theoretical Biology of Cells



Tatsuo SHIBATA

Position Title, Affiliated Department : Fellow, MIMS; Professor, Department of Mathematical and Life Sciences, Graduate School of Science, Hiroshima University

Specialized Field, Academic Degree : Mathematical Modeling and Analysis, Ph.D.

Research Description : Mathematical study of structure formation, locomotion and information processing of living organisms

Research Outline

A cell is a very small system of the size around one to several dozen micrometers, and is still equipped with many of the functions necessary for a living organism, such as responding to changing environment and remembering what is required. With the development of experimental technique, more has become known about the dynamic processes, accompanied with **fluctuations**, of structural formation, information processing, and performing functions within a cell. With the development of molecular biology, a vast amount of information has been accumulated about molecules and their reactions that constitute the cellular processes. In order to shed light on the mechanisms underlying dynamic processes of a cell, there is an increasing need for data analyses, mathematical models, and theoretical analyses, which integrate these pieces of information and apply knowledge of mathematical sciences to highly quantitative experiments in a comprehensive manner.

Recently, there have been many reports that spatiotemporal structural formation is promoted in a cell with mechanisms similar to reaction-diffusion systems. These include temporal oscillations, spatial patterns, and multistability, each of which is responsible for important functions in the respective contexts. As cell-level reactions are highly probabilistic, these mechanisms for structural formation must be robust against probabilistic noises. On the other hand, the mechanisms bring variety of behaviors of a cell by amplifying the probabilistic nature of the elementary steps to the macroscopic level. The researcher is studying how these two seemingly contradictory features are made possible through analysis of fluorescent image data of a single cell and development and analysis of mathematical models.

Embryogenesis is a process to activate the reaction program within a cell accurately to generate, from a single cell, many cells of various types and form structures that are spatially harmonious. The cell-level processes for expression of genes and protein functions exhibit stochastic fluctuations, and it is a big issue in developmental biology to identify the degree of fluctuations and mechanisms for controlling them. In order to approach the theme from both experiments and theories, a method for temporally monitoring functions of genes in the early stage of development has been developed for analysis jointly with the group of Professor Takashi YAMAMOTO in charge of Molecular Genetics and Life Sciences, Department of Mathematical and Life Sciences, Graduate School of Science, Hiroshima University.

Fusion of Mathematical and Life Sciences



Joe Yuichiro WAKANO

Position Title, Affiliated Department : Fellow, MIMS; Associate Professor, Organization for the Strategic Coordination of Research and Intellectual Property, Meiji University

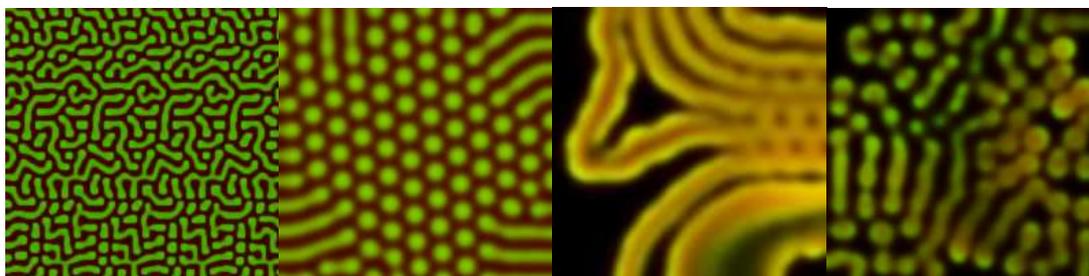
Specialized Field, Academic Degree : Mathematical Biology, Ph.D.

Research Description : MMA of Macrobiology and Ecosystems

Research Outline

The public goods game is a game to choose whether or not to invest in public goods owned by a plurality of players. Many phenomena take the form of a public goods game, from production of antibiotics by bacteria to environmental issues in the human society. The researcher, jointly with C. Hauert of the University of British Columbia, M. Doebeli of the University of British Columbia, and M. Nowak of Harvard University, has studied the ecological public goods game. In a public goods game, cooperation (investment in public goods) tends to evolve when the number of players is limited. Classical studies were focused on the case where the number of players is kept constant. In the ecological public goods game, the number of players is considered to be a function of "the level of investment in public goods for the group as a whole." This is because it is considered that, in a society where more individuals cooperate, the population density becomes high, and as a result, the expected number of individuals sharing the public goods increases. In these circumstances, the researchers demonstrated that a polymorphic population is evolutionarily realized in which only part of the group of individuals invest in public goods [1]. The spatial distribution of individuals and public goods was also analyzed through modeling with reaction-diffusion equations. Accordingly, it was demonstrated that various patterns arise, including the stripe pattern and spatiotemporal chaos (see figure below).

An example of spatiotemporal distribution of cooperation and deceit strategies (brightnesses of red/green represent densities of collaborators and defectors)



[1] Hauert C, Wakano JY & Doebeli M (2008) Ecological Public Goods Games: cooperation and bifurcation. *Theoretical Population Biology* 73:257-263

Various Issues Concerning Networks



Toshikazu SUNADA *Mathematical Analysis Group Leader*

Position Title, Affiliated Department : Fellow, MIMS; Professor, School of Science and Technology, Digital Content Studies and the Connective Humanities, Meiji University

Specialized Field, Academic Degree : Discrete Geometrical Analysis, Ph.D.

Research Description : Analysis of Network System

Research Outline

Networks, abstract figures consisting of points and lines, which are also called graphs, are used in various fields of mathematical sciences as mathematical models representing natural and social phenomena. The concept of networks is in use in a wide range of fields: for example, in economics, the Leontief model that describes the supply/demand relationship among a plurality of industries (interindustrial relations); in engineering, electrical and logical circuits; in information science, Ramanujan graphs modeling effective networks and the Turing machine that is an abstracted computer; in crystallography, crystal lattices that model action of force between atoms; and in mathematical physics, the theory of quantum walks that has been developing rapidly in recent years. The current study focuses on discrete geometric analysis for analyzing discrete structures using geometric methodology developed in global analysis to formulate the basic theory on various issues concerning networks and practical applications thereof.

In the study of the Ramanujan graph to date, I formulated an analogue of the Riemann hypothesis for the Ihara zeta function, and constructed a discrete model for Schroedinger operators under periodic magnetic fields [2]. Further, in the study of crystal lattices, it was identified that a crystal lattice called K4 is the sole mathematical "relative" of diamonds and graphenes [1]. As for the K4 lattice, its possible physical properties are currently being studied in detail with the use of first-principle calculation, and it is suggested that it may be synthesized as an Sp2 carbon crystal. If the synthesis is successful, the K4 lattice is expected to be used in various applications as a crystal having metallic properties [3].

Currently, I am studying asymptotic behavior of quantum walks on crystal lattices and relationships between Cayley graphs associated with discrete groups and Ramanujan graphs. In the study of Ramanujan graphs, it is urgently required to formulate the "arithmetic" subgroups of free groups, and for this end, it is essential to identify the relationship between free groups and quadratic forms. In the study of quantum walks, which are the quantum version of random walks, appropriate formulation and generalization are being sought from the viewpoint of discrete geometric analysis.

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- [2] T. Sunada, Discrete geometric analysis, *Proceedings of Symposia in Pure Mathematics* (ed. by P. Exner, J. P. Keating, P. Kuchment, T. Sunada, A. Teplyaev), 77 (2008), 51-86.
- [3] M. Itoh, M. Kotani, H. Naito, T. Sunada, Y. Kawazoe, and T. Adschiri, New metallic carbon crystal, *Phys. Rev. Lett.* 102 No.5 (2009)

For Mathematical Clarification of Nonlinear Non-equilibrium Phenomena



Masayasu MIMURA *Leader of all research projects*

Position Title, Affiliated Department : Director, MIMS; Professor, School of Science and Technology, Meiji University

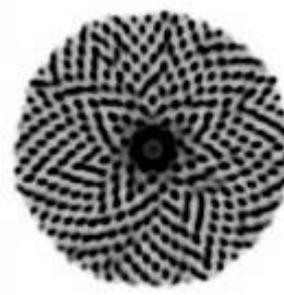
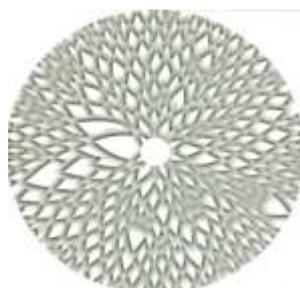
Specialized Field, Academic Degree : Mathematical Modeling and Analysis (MMA), Ph.D. (Kyoto Univ.)

Research Description : MMA of Nonlinear Non-equilibrium Phenomena

Research Outline

In the natural world, we experienced a major paradigm shift in the last half a century. In the words of physics, it was a major shift in the view of nature from static thermodynamics of systems in equilibrium to dynamic thermodynamics of systems in non-equilibrium states, particularly those based on nonlinear, non-equilibrium systems. In order to clarify various phenomena in the nature including life phenomena, the keywords "nonlinear, non-equilibrium" are essential. Such activities as autonomous development of rhythms, patterns and shapes formed in a self-organized manner, singularities such as explosion and condensation are all main players on the stage. These phenomena all have true nonlinearity as the intrinsic factor, and it should be clear that without the clarification of nonlinearity, these phenomena cannot be understood. These nonlinear phenomena are observed in various fields of natural sciences including mathematics, physics, even chemistry, biology, and engineering, and the word "nonlinear" is commonly seen in mathematics, too. However, no studies have been established so far to focus on these phenomena from the viewpoint of "mathematics of non-equilibrium systems," because it is too large a theme to approach from the field of mathematics only. Nevertheless, a mathematical science methodology called mathematical sciences based on modeling and analysis has gradually been established, thanks to collaboration with other fields, rapid development of computers, and feasibility to pursue numerical approaches and pure mathematical theoretical approaches from the complementary viewpoint. The research activities in charge of the project for FY 2008 are included:

- (1) Mathematical analysis of smoldering combustion model under nonlinear, non-equilibrium conditions;
- (2) Development of singular limit methods of reaction-diffusion equations;
- (3) Pattern formation arising in nonlinear, non-equilibrium reaction-diffusion equations; and
- (4) Spatial segregation theory of competitive and diffusion equations.



Theoretical Foundation of Combinatorial Optimization and Application to Mathematical Sciences Based on Modeling and Analysis



Hisao TAMAKI

Position Title, Affiliated Department : Fellow, MIMS; Professor, School of Science and Technology, Basic Science and Technology, Meiji University

Specialized Field, Academic Degree : Theory of Computation, Ph.D.

Research Description : Computation and Theory of Algorithms

Research Outline

Combinatorial optimization was studied broadly from theoretical foundation to applications.

For theoretical approaches, the researcher emphasizes studies on branch decomposition of graphs as the basis for very large-scale neighborhood local search. The overall structure of the study is as follows. There is the local search or interactive improvement method as a commonly used method for combinatorial optimization. In this approach, a tentative solution is improved interactively in expectation of obtaining a solution close to the optimal solution. In each of the steps for improvement, the group of candidates for solution that are improvements to the tentative solution is called the neighborhood. If there are no solutions that are better than the tentative solution, the local search is completed. Naturally, the solution obtained in this method is only locally optimized, and is not necessarily the true optimal solution. The idea of very large-scale neighborhood is to use a huge neighborhood (usually exponential to the input value), in order to reduce the possibility of ending up with a globally non-optimal local optimal solution.

In the very large-scale neighborhood local search, there arise the sub-problems of optimization in the very large-scale neighborhood. The researcher has been seeking the approach of designing the very large-scale neighborhood as the "problem on a graph with branch decomposition of small width," and has achieved some successes, for example, in the traveling salesman problem. In this approach, it is very important to solve the optimal branch decomposition of the graph. This problem is known to be NP-hard for general graphs. For planar graphs, however, Seymour and Thomas have demonstrated that it can be solved in $O(n^4)$ time. The researcher, together with Qianping Gu, improved the algorithm so that it can be solved in $O(n^3)$ time. This study appeared in 2008 ACM Transactions on Algorithms. Furthermore, the researcher, together with Qianping Gu, have achieved some theoretical successes, such as approximation algorithms for branch decomposition of planar graphs and a new upper bound on the ratio of the branch width of a planar graph to the size of its maximum grid minor. These results will be reported in international conferences in FY 2009.

In applications, the researcher has been working on computer science applications such as for graph drawing. The researcher has also been studying applications in mathematical sciences based on modeling and analysis, particularly for simulations. A potential candidate for the applications was identified during a discussion session held toward the end of the fiscal year with the study group of Mr. Kanya KUSANO of the Earth Simulator Center, who is also one of the researchers at GCOE responsible for promotion of its projects. It is the problem of estimating the solar magnetism from solar light observation data. The magnetic field vector estimated from polarization of the observed light has a 180-degree uncertainty, and the problem of removing this uncertainty from the standpoint of global consistency can be formulated as the 0-1 optimization problem. The researcher has started studying whether the very large-scale neighborhood approach is effective for the solution of this problem.

Understanding and Forecasting Solar Flare Eruption with Multi-scale Simulation



Kanya KUSANO *Simulation Group Leader*

Position Title, Affiliated Department : Fellow, MIMS; Visiting Professor, School of Science and Technology, Meiji University

Specialized Field, Academic Degree : Simulation Science, Ph.D

Research Description : Modeling and Simulation of Large-scale Cascade Systems

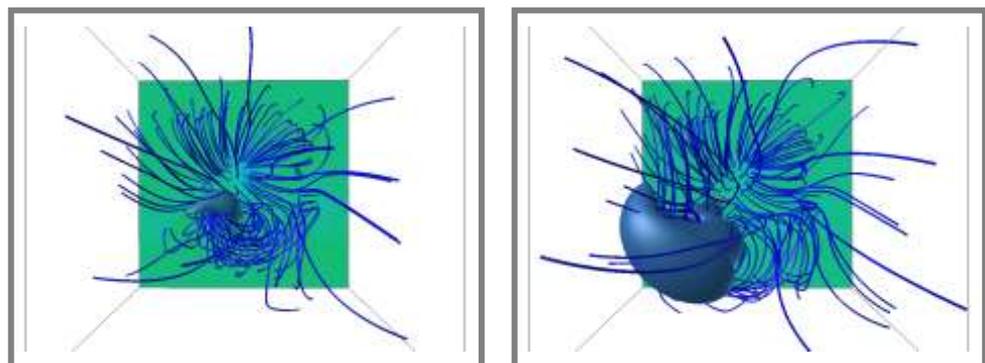
Research Outline

Modeling of complex phenomena, to which elementary steps with differences in spatial and temporal scales are related in a compounded manner, is considered as an important study subject in many fields. In particular, understanding of multi-scale processes is important in the study of plasma, as fine movements of electrons and ions and magnetohydrodynamic phenomena of various scales interact with each other in a very complicated manner. Accordingly, we have studied and developed a new simulation that links different physical layers, in order to understand and forecast emergent phenomena such as solar flares and coronal mass ejections in the large-scale plasma system comprising the sun and earth's magnetosphere.

Eruptive phenomena in the sun-earth interlocked system occur due to release of free energy accumulated in the solar surface magnetic field and propagation of shock waves caused by the energy release. Accordingly, we first built models corresponding to the solar surface, solar corona, and interplanetary space, respectively, and then tried to reproduce the series of phenomena by interlinking the models. Furthermore, the researchers obtained an equilibrium solution of the solar coronal magnetic field as the boundary value problem, based on the solar surface magnetic field conditions observed by the solar science satellite "Hinode," and succeeded, for the first time in the world, in reproducing the solar flare eruption of December 2006 in simulation. In the simulation, the coronal magnetic field was destabilized through virtual solar surface movement. Accordingly, through numerical experiments using various virtual flows, the possibility of forecasting solar flare occurrences was sought by quantitatively assessing the degree of destabilization due to the flows in the coronal magnetic field. As a result, it was demonstrated that use of data observed close to the time of the actual flares reproduced larger eruptions. This implies that it may be possible to forecast large-scale eruptions in the space by means of numerical models based on observed data, and that this could help contribute to understanding complex phenomena as well as to sophistication of space weather forecast.

Figure:

Structural changes in magnetic field lines and shock wave front (shown in blue constant-height surface) immediately after a solar flare (left) and thereafter (right), reproduced in simulation based on observed data.



Visualization of Limit Set of High-dimensional Kleinian Groups



Kazushi AHARA

Position Title, Affiliated Department : Fellow, MIMS; Associate Professor, School of Science and Technology, Basic Science and Technology, Meiji University

Specialized Field, Academic Degree : Mathematics, Ph.D.

Research Description : Development of software for researchers of topology

Research Outline

Kleinian groups are a properly discontinuous subgroups of the isometry group of the three-dimensional hyperbolic space, and are also given as subgroups of Moebius transformation groups on a 2-dimensional sphere. We study those of higher dimensional case, i.e., subgroups of the isometry group of the four-dimensional hyperbolic space or subgroups of Moebius transformation groups on a 3-dimensional sphere, and we call them 4-dimensional Kleinian groups, which are the main interest.

Kleinian groups are represented in $SL(2, \mathbb{C})$ (2-dimensional special linear group), while the high-dimensional Kleinian groups are represented as 2-by-2 quaternionic matrices. All elements of the Kleinian groups are classified into three types: elliptic, loxodromic, and parabolic. In the 4-dimensional Kleinian groups case, it is known that these types are further sub-classified into "simple" and "compound," and hence, there are six types in total. This classification can be described with conditions about some conjugacy invariants of the matrix representation.

The set of accumulation points of the orbit of a point of the Kleinian groups is called a limit set. This is the set found on the infinity point set of the 4-dimensional hyperbolic space (which is homeomorphic with 3-dimensional sphere), and have "complicated" figure having, in many cases, a fractal structure. In studies of Kleinian groups, we restrict on generators (we called the restriction the slice) and we study conditions when we obtain Kleinian groups and shapes of the limit set. In the case of (3-dimensional) Kleinian groups, the limit sets are fractal "lines" on the 2-dimensional sphere. In some cases, they are the sum of an infinite number of circles in contact with each other (Apollonian gaskets). In the case of quasifuchsian groups, there arises shapes which is homeomorphic with a circle (with a fractal structure.) For 4-dimensional Kleinian groups, both "linear case" and "spherical case" are known. Some are a single fractal polygonal line, and some have the shape of an infinite number of spheres in contact with each other. Some are fractal surfaces that are homeomorphic with 2-dimensional spheres.

Since the limit set is a set of accumulation points of an orbit by definition, graphically it might be drawn only in pointwise. We apply theorems of mathematics to identify the structure of the limit set such as "linear" or "planar" or "spherical." Visualization of the limit set is therefore nothing but mathematical analysis of structures of 4-dimensional Kleinian groups.

Looking at Mechanisms for Generating Complex Patterns Using Simulations



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Position Title, Affiliated Department : Fellow and DST, MIMS; Associate Professor, School of Science and Technology, Basic Science and Technology, Meiji University
Specialized Field, Academic Degree : Mathematical Sciences, Ph.D.
Research Description : Simulation Aided Analyses

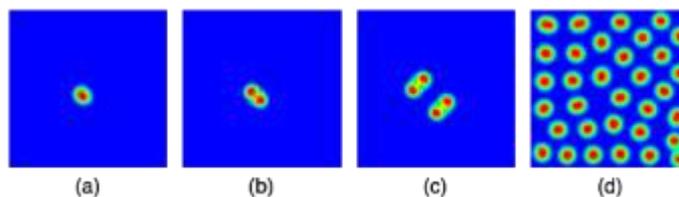
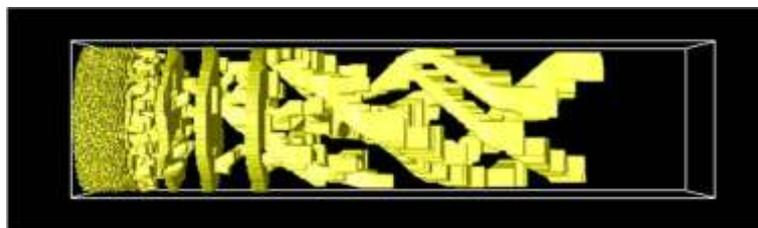
Research Outline

By what mechanisms are the complex patterns, observed in the natural world, generated? The way living organisms create patterns is really awesome, but complex patterns that attract people are also observed among non-living things. For example, the pattern called Liesegang disks, which is observed in some chemical precipitation reaction systems in the gel, have long interested researchers as beautiful mathematical laws are observed in the layout. Recently, it was found that the transition between the orderly and complex patterns (the latter including arborization) depends on the properties of the gel, which promoted new developments in the study. The researcher is trying to clarify the basic mechanism of the chemical precipitation reaction systems, using the powerful weapons of modeling and simulation.

Also, in an autocatalyst reaction diffusion system that is part of a complex chemical reaction system, a pattern of self-reproduction, which is similar to cell divisions, is known to be derived as a solution of the equations. The researcher is trying to approach the basic mechanism responsible for these complex spatiotemporal patterns by means of simulation and computer-aided analysis.

The researcher is working hard on these studies, believing that modeling, as well as simulation and analysis using computers, will one day be the methodology for clarifying complex phenomena, including life phen

Simulation results of chemical precipitation reaction system in 3-dimensional gel



Temporal development of solution for self-reproducing pattern

Analysis of Spatial Pattern Observed in Reaction Diffusion Equation Systems



Kota IKEDA

Position Title, Affiliated Department : Research Fellow, MIMS; Research Promoter, "GCOE SPD", Meiji University

Research Description : Reaction-diffusion equation, Pattern formation

Research Outline

In order to reproduce spatial patterns observed in various phenomena, many reaction-diffusion systems have been proposed. One of the important study themes in the field of the pattern formation problem is to clarify mathematical properties associated with the spatial patterns, and can be divided into two major sub-themes: obtaining necessary conditions to reproduce the spatial pattern; and identifying equation systems that can reproduce complex spatial patterns.

An example study belonging to the first sub-theme is the well-known mathematical result that no reaction-diffusion equations having a single unknown function in the convex domain have a stable and spatially inhomogeneous stationary solution. From this result, it can be known that a simple equation is not sufficient to reproduce spatial patterns. On the other hand, a reaction-diffusion equation system having two unknown functions can reproduce complex spatial patterns. However, because the spatial patterns thus reproduced are so complicated that, in order to drive mathematical studies, it is more appropriate to give some appropriate conditions and look at simpler spatial patterns. For this reason, the researcher took up a shadow system corresponding to a condition that the ratio of the diffusion coefficient is very large and demonstrated instability of the multiple spot. The study focused on generalized systems, and the result is therefore general.

The researcher is also studying the second sub-theme. Specifically, the researcher is working on mathematical studies of model equations for combustion proposed by Dr. Masayasu MIMURA, the joint researcher. By using the system, spatial patterns, which are observed in laboratory tests but have not been available in exact analyses, can be reproduced. In detail, by changing the parameter values of the system, a variety of combustion surfaces are generated, including even, wave-form, and finger-like combustion surfaces. Reflection of combustion is also observed. These spatial patterns have some common characteristics, and the 1-dimensional traveling wave solution was expected to play an important role. Accordingly, it was essential to construct the 1-dimensional traveling wave solution, and the researcher was successful to do so. Further, the researcher also demonstrated that the traveling wave solution is stable under appropriate conditions. Generally, similar results stand for monostable reaction-diffusion systems called excitable systems. This result can be regarded as new. The objective of further study is to advance the current research and construct solutions that reproduce the various spatial patterns described above.

Theoretical Study of Evolution of Learning



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Position Title, Affiliated Department : Research Fellow, MIMS; Research Promoter, "GCOE PD", Meiji University
 Research Description : Theoretical Anthropology: Evolution of learning ability, Sexual selection

Research Outline

In the theoretical study of evolution of learning, the focus was on whether "a certain behavior" evolves to be generically inherited or to be acquired through individual or social learning (Boyd and Richerson, 1985, 1988, 1995; Rogers, 1988; Feldman et al., 1996; Henrich and Boyd, 1998; Wakano et al., 2004; Aoki et al., 2005; Wakano and Aoki, 2006, 2007; Nakahashi, 2007; Enquist et al., 2007; McElreath and Strimling, 2008; Aoki and Nakahashi, 2008). Here, individual learning means groping for a correct behavior by oneself, while social learning means learning a behavior from other organisms. If an organism learns from the parents, it is called vertical transmission; if from a random member of the parental generation, oblique transmission; and from a random member of the same generation, horizontal transmission. Sometimes, social learners do not learn from a random member of the population but from the majority, from successful organisms, or from organisms in high positions.

On the other hand, recent empirical researches revealed that there are positive correlations between observation frequencies of individual and social learning activities among species (Reader and Laland, 2002) and among organisms (Bouchard et al., 2007). However, in prior studies of evolution of learning that focused on strategies for acquiring a behavior, the sum of the weights assigned to each strategy must be one due to the restriction of the models. This means that there exists a trade-off between social and individual learning in these models, which cannot explain the positive correlations in empirical researches. Accordingly, in order to address the problem, the researcher built a new model for analyzing how many times individual and social learning activities evolve to be performed.

In the analysis, the researcher used the ESS (evolutionarily stable strategy) method (Maynard Smith 1982) to identify which strategy is evolutionarily stable. Also, the researcher determined analytically if the strategy found is actually achieved as a result of evolution, i.e., if it is a continuously stable strategy (CSS) (Eshel, 1983). Further, the researcher ran numerical simulations for competition of various strategies to determine if the simulation results are consistent with those found analytically.

These analyses of how many times individual and social learning activities evolve to be performed revealed that there are cases where the numbers of individual and social learning activities have positive correlations. We can consider that the more an organism performs the learning activities, the more his learning activities are observed by observers. The study results thus explain the positive correlations between observation frequencies of individual and social learning activities in empirical researches. Further, the current study made it possible to theoretically forecast under what circumstances the number of individual and social learning activities, i.e., the observation frequencies of respective learning activities, will have positive or negative correlations.